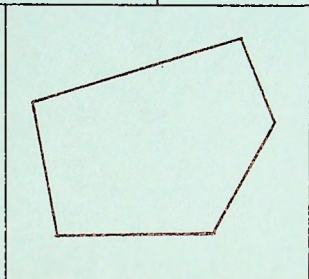


**S**

## The Bicentennial Star

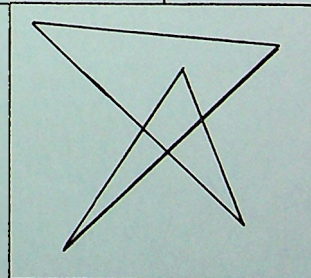
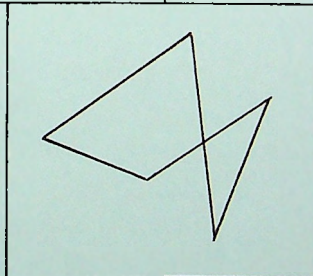
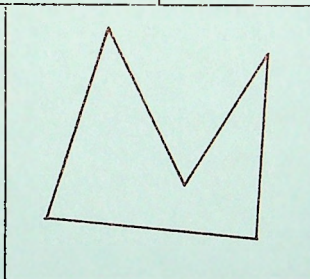


Five points are picked at random within the unit square. When connected in the order in which they are selected, they may form a 5-pointed star, as at the example labelled S.

Or, they may form other shapes, as in the other examples. There are also various degenerate cases in which successive points are collinear.

The star shape may be defined as having a pentagon in its center, with distinct triangles contiguous to the five sides of the pentagon.

Problem: What are the chances of the process producing the star? What is the logic of distinguishing the star from the other shapes, other than by plotting and inspecting by eye?



PROBLEM 119



Log 36	1.556302500767287265017533595959216671936637491305609
ln 36	3.583518938456110001624954716761404545445981384366009
$\sqrt[3]{36}$	3.301927248894626683874609952409084956846884644318493
$\sqrt[5]{36}$	2.047672511079219296212837356328621875496219185196690
$\sqrt[10]{36}$	1.430969081105255501045224413143116904972649939661282
$\sqrt[100]{36}$	1.036485008681194204461773260833159530952951610747034
$e^{36}$	4311231547115195.227113422292856925390788863616780347 730770167843912372047055558562843132
$36$	789578687047901181.0888160796210953238167966682574019 3544190769936945166565878088145448
$\tan^{-1} 36$	1.543025690201475582748793062186532122271951805052459

The price schedule as of June 1, 1976 is as follows:

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# Factorials

The sequence at the left is a well-defined function. It consists of the low-order non-zero digits of consecutive factorials, shown through factorial 114. The sequence does not appear to repeat within the first 210 terms.

- (A) Will the sequence cycle (that is, repeat)?
- (B) If so, what is the cycle length?
- (C) Will the digits 2, 4, 6, and 8 appear equally often? The counts for terms 2 through 210 are:

2	56
4	52
6	49
8	52

1	4
2	6
6	6
4	6
2	2
2	6
4	4
2	6
8	6
8	2
8	6
6	4
8	8
2	8
8	6
6	2
8	4
2	4
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2	4
8	4
6	8
6	4
2	6
6	8
4	8
2	6
2	8
4	2
2	2
8	2
4	4
4	4
8	2
8	8

# Contest 5

In the pattern at the right, the consecutive integers lie above the line. The arrows below the line mark the positions of the prime numbers.

The following sieve is to be implemented.

With the conditions as indicated above, the arrows denote those numbers which are to be deleted. The reference point number (zero) is noted. The numbers remaining are then written on the line above, starting at the reference position, as shown one line above. Again, the arrows indicate which numbers are to be crossed off. As the process continues, the numbers at the reference point form the sequence: 0, 1, 4, 9, 16, 26, 39, 56, 78, 106,...

The longest extension of the sequence (prepared by computer, of course) will receive the \$25 prize in this, our 5th contest. All entries must be received by POPULAR COMPUTING by May 31, 1976.

REFERENCE POINT

PROBLEM 121

39	56	69	77	78	84	94	100	105	106	1	124	125	126	133	140	142	145	152	56	...
26	39	49	55	56	60	69	74	77	78	84	93	94	95	100	105	106	110	115	119	...
16	26	33	38	39	42	49	52	55	56	60	68	69	70	74	77	78	80	84	88	...
9	16	21	25	26	28	33	36	38	39	42	48	49	50	52	55	56	57	60	64	...
4	9	12	15	16	18	21	24	25	26	28	32	33	34	36	38	39	40	42	45	...
1	4	6	8	9	10	12	14	15	16	18	20	21	22	24	25	26	27	28	30	...
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	...



# Solution to the Q Problem

PC36-5

The Q problem (No. 82) appeared on the cover of issue 24. A 10 x 10 array of cells is identified from 0 to 9 across the top and down the side. Each of the 100 cells then has a unique identifying address, from 00 to 99. The notation is that of a matrix; for example, cell 68 refers to row 6, column 8.

The problem was the following: to enter the array at cell 00 and trace a path through the array, ending at cell 00, with the longest possible trip, measured in terms of squared distances. Each cell is to be visited just once. The array will thus contain all the numbers from 00 through 99, with the numbers pointing to the cell to be next visited. The pattern given in issue 24 had the number 99 in cell 00, so the first leg of the sample trip had a squared distance of  $81 + 81 = 162$  units. At cell 99 there was the number 10 (pointing to row 1, column 0), so the second leg produced a distance of  $64 + 81$  units. The sample pattern given had a squared distance sum of 6026, which had been obtained from considerable hand juggling of numbers, plus computer runs to calculate the necessary sums.

Thomas R. Parkin writes:

"Let us consider the problem in the following way. What is the best possible pairing of the numbers from 00 to 99 such that the total of their squared differences, considered one digit at a time, is maximum, regardless of their location? Then, after answering that question, let's see if we can construct that arrangement. The ideally best possible pairing of digits would be obtained as follows:

9 9	9 8	9 7	9 6	9 5	....	0 3	0 2	0 1	0 0
0 0	0 1	0 2	0 3	0 4	....	9 6	9 7	9 8	9 9
<hr/>									
9,9	9,7	9,5	9,3	9,1	....	9,3	9,5	9,7	9,9

Thus, a 9 is opposite each zero, a 1 opposite each 8, and so on. The squared differences will then have 40 each of the square of 9, 7, 5, 3, and 1 for a total of 6600.

This ideal is not realizable. We could approach it with the pattern shown at A, which has a squared total of 6130 (already more than the 6026 given in issue 24). But we still cannot put the numbers in the cells in that order, because of the directional nature of the coordinates; we would be going down and across to an inevitable intersection. However, we notice that interchanging a few numbers doesn't change the sets of differences, if we are careful. So, let's order the numbers so that we proceed down the columns from top to bottom and across the rows from right to left. We quickly develop array B which has a sum of 6130.

00		10	
	9,9		7,9
99		89	
	9,8		7,8
01		11	
	9,7		7,7
98		88	
	9,6		7,6
02		12	
	9,5		7,5
97		87	
	9,4		
03		.....	
	9,3		
96		44	
	9,2		1,1
04		55	
	9,1		1,0
95		45	
	9,0		1,1
05		54	
	9,1		1,2
94		46	
	9,2		1,3
06		53	
	9,3		1,4
93		47	
	9,4		1,5
07		52	
	9,5		1,6
92		48	
	9,6		1,7
08		51	
	9,7		1,8
91		49	
	9,8		1,9
09		50	
	9,9		
90			

A





Now, let's see if we can improve that. We notice that zero values come from the transitions from the bottom of a column to start of the next column. What if we zig-zagged down instead of returning to the top of the column each time? The pattern should be: top to bottom down the first column, bottom to top in the last column; bottom to top in the second column, top to bottom in the next to last column; and so on, in a zig-zag pattern. This yields pattern D, for a total of 6454. Note that we have converted 4 zeros to 4 nines in one coordinate.

Well, I think we are about done. Of course, any of millions of permutations and rotations of this pattern will also generate 6454. I believe that this is about as far as we can go. Note that we are only 146 short of the best possible sum of squares of sets of differences." □

99 98 97 96 95 00 04 03 02 01

89 88 87 86 85 94 93 92 91 90

79 78 77 76 75 84 83 82 81 80

69 68 67 66 65 74 73 72 71 70

59 58 57 56 55 64 63 62 61 60

49 48 47 46 45 54 53 52 51 50

39 38 37 36 35 44 43 42 41 40

29 28 27 26 25 34 33 32 31 30

19 18 17 16 15 24 23 22 21 20

09 08 07 06 05 14 13 12 11 10

99 98 97 96 95 00 83 93 81 91

89 88 87 86 85 94 73 92 71 90

79 78 77 76 75 84 63 82 61 80

69 68 67 66 65 74 53 72 51 70

59 58 57 56 55 64 43 62 41 60

49 48 47 46 45 54 33 52 31 50

39 38 37 36 35 44 23 42 21 40

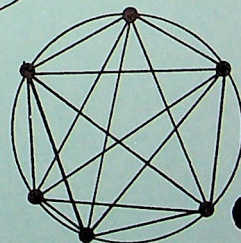
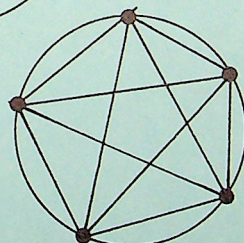
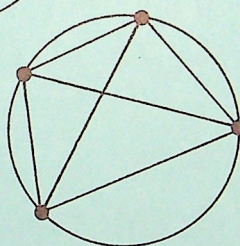
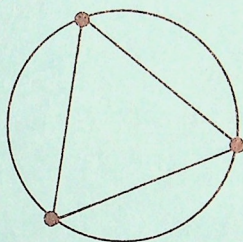
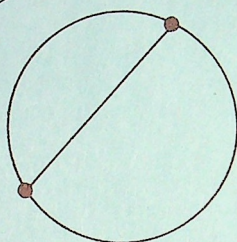
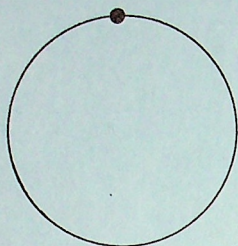
29 28 27 26 25 34 13 32 11 30

19 18 17 16 15 24 03 22 01 20

09 08 07 06 05 14 04 12 02 10



The six circles shown on this page have  $N$  points marked on their circumferences, with every pair of points connected by straight lines. We count the number of parts each circle is thus cut into:



number  
of  
points

1	1
2	2
3	4
4	8
5	16
6	?

number of  
parts of  
the circle

PROBLEM 122

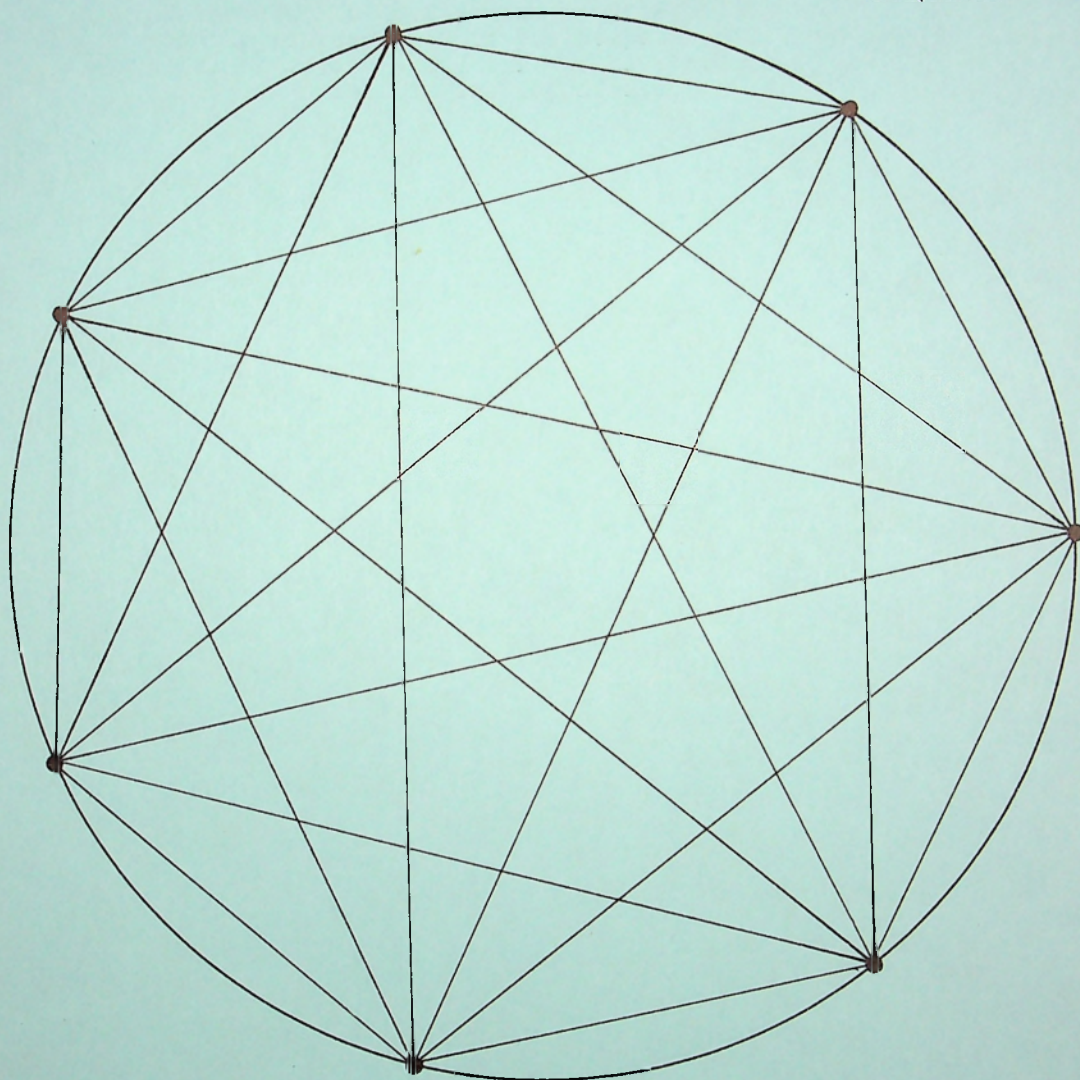
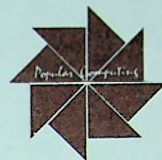
## Circle Partitioning

What is the value of this function for  $N = 6, 7, \dots, K$ ?

The figure for  $N = 7$  is given on the facing page.

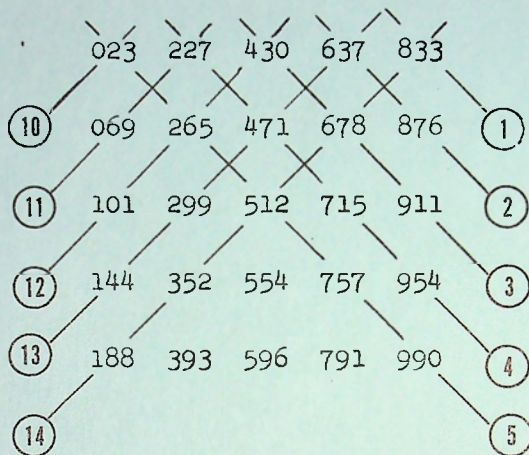
Notice that when  $N$  is even, the points should not be placed at regular intervals around the circumference, lest the polygon at the center be lost.





CIRCLE PARTITIONING: 7 divisions of  
the circle's circumference--count the  
number of parts the circle is cut into.

# Contest 2 Results



The original pattern given in issue No. 31, showing the numbering scheme for the diagonals

637	299	101	023	512
352	265	227	596	990
144	188	471	833	876
069	430	791	757	678
554	954	911	715	393

Daughterity's pattern

393	352	144	023	554
299	265	188	430	990
101	227	471	791	911
069	596	833	757	715
512	954	876	678	637

Beeler's pattern

Our second contest (Average Variances, Problem 106) appeared in issue No. 31, October 1975. The problem was to arrange twenty five specific numbers in a 5 x 5 array so that the average of the variances in the positive direction (numbered 1, 2, 3, ..., 9) would differ from the average of the variances in the negative direction (numbered 10, 11, 12, ..., 18) by the greatest amount.

Two entries with slightly differing patterns had identical results: 108877.30079. The joint winners, both richer by \$25, are:

Walter C. Daughterity, ECRM, Inc., Bedford, Massachusetts

Mike Beeler, BBN Div. 46, Cambridge, Massachusetts



# HP-65 Users Club

PC36-11

The pocket programmable calculator began with the Hewlett-Packard HP-65 toward the end of 1973. In June, 1974, the HP-65 Users Club started, under the capable direction of Richard J. Nelson (2541 West Camden Place, Santa Ana, CA 92704). The club has expanded to include the HP-55 and HP-25, and the members are currently considering including also the Texas Instruments' SR-52.

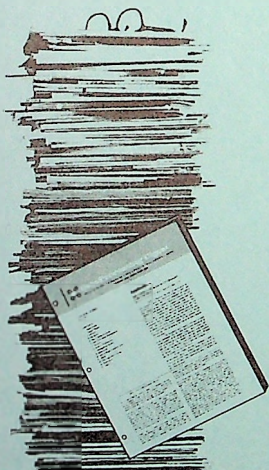
Members contribute \$12 per year (U.S.) to defray the cost of the voluminous notes published each month. In addition to listings of programs, the notes include a wealth of information about the machines, much of it unobtainable from the manufacturer. For example:

1. A program to develop the 36-digit product of two 18-digit integers.
2. Certain keystrokes, switch operations, or programming sequences will cause undocumented calculator behavior. All known crashes, halts, or errors have been discussed in 65 NOTES.
3. A practical keyboard overlay and unique encoding technique allows keying the alphabet and special symbols into the machine, paving the way for word games and cryptographic schemes.
4. A printer was designed to attach to the 65, and the complete design was printed in the NOTES.
5. The December 1975 issue does deal with the SR-52 and includes comparisons between the SR-52 and the HP-65, particularly as to the speed of operation.
6. A lively discussion has been going on about What Comes Next? Both Hewlett-Packard and Texas Instruments have been provided with extensive notes on how to design their next machine.



"The Turbo-Encabulator in Industry" was written by J. H. Quick, B.Sc., in 1944 and was first published in the (London) Institution of Electrical Engineers' Students Quarterly Journal. It was published in this country in the February, 1946 Industrial Bulletin of Arthur D. Little, Inc. In the computing field, it appeared in the June 1, 1955 issue of Computing News.

Mr. Quick's brilliance pre-dated computers, of course, but the satire applies equally well to the literature of our field. After 32 years, the new generation deserves to know about nofer trunnions, tremle pipes, and dingle arms.



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## The TURBO-ENCABULATOR in INDUSTRY

FOR a number of years now work has been proceeding in order to bring to perfection the crudely conceived idea of a machine that would not only supply inverse reactive current for use in unilateral phase detractors, but would also be capable of automatically synchronizing cardinal grammeters. Such a machine is the "Turbo-Encabulator." Basically, the only new principle involved is that instead of power being generated by the relative motion of conductors and fluxes, it is produced by the nodal interaction of magneto-reluctance and capacitive directance.

The original machine had a base-plate of prefabricated aluminum, surmounted by a malleable logarithmic casing in such a way that the two main spurring bearings were in a direct line with the penametric fan. The latter consisted simply of six hydroscopic marzlevanes, so fitted to the ambifacient lunar waneshaft that side fumbling was effectively prevented. The main winding was of the normal lotus-o-delta type placed in pancardemic semi-bowoid slots in the stator, every seventh conductor being connected by a non-reversible tremie pipe to the differential girdle-spring on the "up" end of the grammeters.

Forty-one manestrically spaced grouting brushes were arranged to feed into the rotor slip-stream a mixture of high S-value phenylhydrobenzamine and five per cent ruminative teryliodohexamine. Both these liquids have specific pericosities given by  $P = 2.5C, 6.7$  where  $n$  is the diathetical evolute of retrograde temperature phase disposition and  $C$  is Cholmondeley's annular grillage coefficient. Initially,  $n$  was measured with the aid of a metapolar refractive pilfrometer (for a description of this ingenious instrument, see L. P. Rumpelverstein in "Zeitschrift für Elektrotechnische-Donoet-litze," vol. vii), but up to the present date nothing has been found to equal the

transcendental hopper dadascope. (See "Proceedings of the Peruvian Nitrate Association," June 1914.)

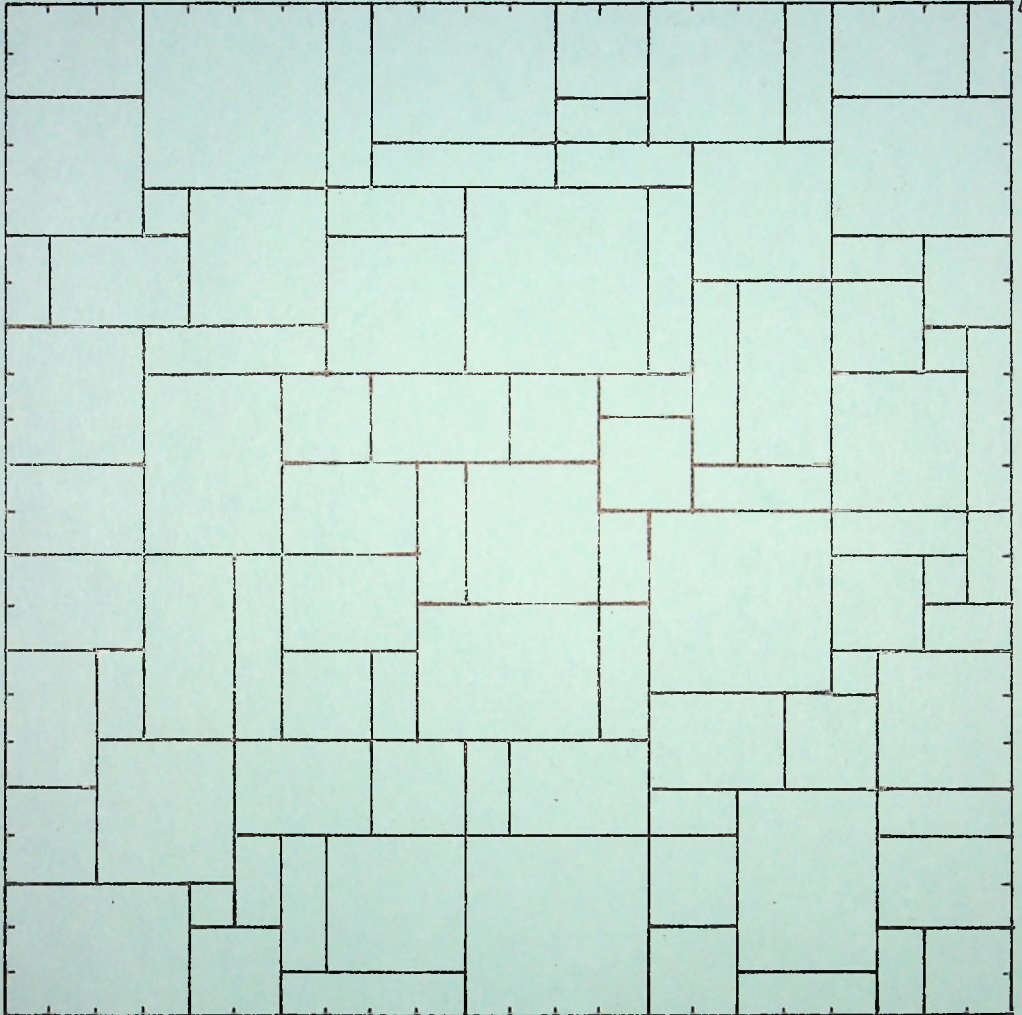
Electrical engineers will appreciate the difficulty of nubing together a regurgitative pugwell and a supramitive wenne-sprocket. Indeed, this proved to be a stumbling-block to further development until, in 1942, it was found that the use of anhydrous nangling pins enabled the kryptonastic bolling shims to be tankered.

The early attempts to construct a sufficiently robust spiral decommutator largely failed because of a lack of appreciation of the large quasi-piestic stresses in the gremlin studs; the latter were specially designed to hold the rofit bars to the spam-shaft. When, however, it was discovered that wending could be prevented by a simple addition to the jiving sockets, almost perfect running was secured.

The operating point is maintained as near as possible to the h.f. rem peak by constantly fromaging the bitumogeous spandrels. This is a distinct advance on the standard nival-sheave in that no dremcock oil is required until after the phase detractors have remissed.

Undoubtedly, the turbo-encabulator has now reached a very high level of technical development. It has been successfully used for operating nofer trunnions. In addition, wherever a batescent skor motion is required, it may be employed in conjunction with a deep-drawn reciprocating dingle arm to reduce sinusoidal depletion.

$\uparrow$  x



Shortcut



## Shortcut

A pattern of fields forms a  $22 \times 22$  grid. The fields, of varying rectangular shapes, are fenced off, as shown.

A traveler proceeds from one corner, marked X, to the opposite corner, marked Y. Following the fence lines in any order will take him at least 44 units to go from X to Y.

However, he may cut across any 6 fields diagonally; that is, "kitty corner." These shortcuts must be from corner to corner; that is, he may not enter or leave a field anywhere but at a corner, and must otherwise follow the fence lines.

If he chooses to shortcut a  $4 \times 4$  field, he will shorten his trip by  $(8 - 4\sqrt{2})$  units. A shortcut across a  $2 \times 4$  field, on the other hand, gains him only  $(6 - 2\sqrt{5})$  units.

What path should he follow for the shortest trip?

## Book Review

Computational Mathematics by Gideon Zwas and Shlomo Breuer, Tel Aviv University, University Publishing Projects, Ltd., 28 Hanatziv Street, Tel Aviv 67015 Israel, 230 pages, 6 x 9 soft cover, \$7.

This is a book on numerical computation, a sort of introduction to numerical methods. It covers such topics as series evaluations, numeric integration, roots of linear systems, and the calculation of  $\pi$ . The treatment is low level, clearly aimed at beginners. It is a How To Do It manual, rather than a compilation of derivations and proofs. The style is light and charming, and the authors' enthusiasm for the fun involved in solving problems is evident throughout.



The chapter titled "The Calculation of  $\pi$  to a High Accuracy" is misnamed and flawed. It begins with the method of Archimedes, which was covered carefully in Richard Hamming's article in our issue No. 12. The authors then proceed to algorithms involving arctangent series, but fail to show how the calculations are to be performed to any precision beyond that conveniently available in a language like Fortran. The chapter then goes into the Buffon needle-tossing approach at great length, using a random number generator to simulate the tosses of the needle. They encourage the reader to run their program for 25,000,000 tosses, overlooking the fact that the process is severely limited by the random number generator being used, so that the result is a test of the random number generator (and a poor one) rather than a calculation that could lead to more than a few digits of  $\pi$ .

The reader is taught to calculate a variance according to its definition (the average squared deviation from the mean). They seem to be unaware that, in most such situations, there is also a computational formula that is simpler, more direct, using much less CPU time, and (possibly) yielding more accurate results.

The algorithms developed in the book are presented in narrative style (rather than by flowcharts, pseudocode, or procedural languages). Each algorithm is given a name; some of these are self-evident, such as LOGNAT for a scheme for calculating natural logarithms, but some are obscure, such as SINUS and URV.

The book is intended to be used in parallel with a traditional calculus course, to provide the link between calculus and computing. This it does well, and it may be the first to attempt to introduce calculus students to the equally mysterious art of coaxing numerical results from a machine. The authors had access to a Control Data 6500, a Data General Nova 10, and an IBM 370.

Generally, the book stresses standard techniques for doing numerically what closed-form solutions indicate can be done. It would be greatly strengthened by showing areas in which computation can handle situations for which there is no closed-form solution (e.g., inversion of functions, roots of complicated equations, and certain combinatorial problems). In other words, besides showing the computer as an adjunct to the calculus, it would be nice to show also those areas in which the computer is an entirely new tool.

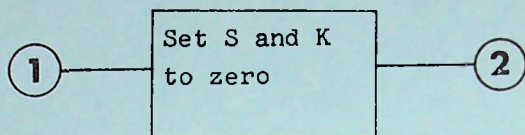
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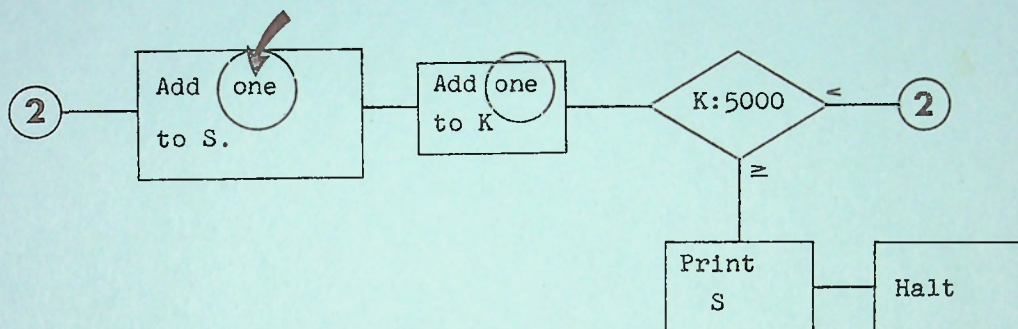
Don't Wait. Why are you waiting? You have lying in the back of your head the idea that some day you are going to do something that will make your friends happy over your success. You can almost hear them saying that they never supposed you had it in you. Your breast often swells with joy as you think of being honored and applauded after you have achieved the great thing. You feel sure that you will be able to rise to the opportunity when it comes. But why are you waiting? A year ago you were waiting for this year; a week ago you were waiting for today. Time is up. Cease waiting. The great thing will never be realized if you keep putting it off. Shape your actions to the present. Do the great thing now--at least make a beginning. Every day that you put it off is a day less of satisfaction and of honor for you.







*Another Way*



The logic outlined in the flowchart should lead to output of 5000 for the sum, S.

However, the two values of one are not the same.

The one added to K is just a plain old integer 1. The one to be added to S can be one of the following:

(A)  $\sin^2 X + \cos^2 X$ , with X chosen at random in some suitable range.

(B)  $\ln \left[ e^{\left( \frac{1}{\cos^2 X} - \frac{\sin^2 X}{\cos^2 X} \right)} \right]$  for any X

(C)  $\frac{\sec^2 X (1 + \csc X) - \tan X (\sec X + \tan X)}{\csc X (1 + \sin X)}$

(D)  $\frac{2\cos 5X \cos 3X + 2\sin 5X \sin 3X}{(\csc X)(\sin 3X - \sin X)}$   $0 < X < 1$

(E)  $\cot^2 X - (\cot 2X)(2\cot X)$

(F)  $32\cos^6 X - 48\cos^4 X + 18\cos^2 X - \cos 6X$

PROBLEM 124

Suppose that we choose one of these 6 ways of expressing unity at random each time that S is to be incremented. What will be the value of S at Halt?